Central Bank Digital Currency with Collateral-constrained Banks

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November 27, 2024

Abstract

We analyze the risks to bank intermediation following the introduction of a central bank digital currency (CBDC). The CBDC competes with commercial bank deposits as the household's source of liquidity. We revisit the result in the literature regarding the equivalence of payment systems by introducing a collateral constraint for banks when borrowing from the central bank. When comparing two equilibria with and without the CBDC, the central bank can ensure the same equilibrium allocation and price system by offering loans to banks. However, to access loans, banks must hold collateral at the expense of extending credit to firms, and the central bank assumes part of the credit-extension role. Thus, in the equivalence analysis, while the CBDC introduction has no real effects on the economy, it does not guarantee full neutrality as it affects banks' business models. In a dynamic model extension, we analyze the effects of an increase in the CBDC and show that the CBDC not only does not cause bank disintermediation or crowd out of deposits but may foster an expansion of bank credit to firms.

JEL codes: E42, E58, G21

Keywords: Central bank digital currency, collateral-constrained bank, equivalence, credit extension

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This paper previously circulated under the title "Unveiling the Interplay between Central Bank Digital Currency and Bank Deposits". We are grateful to Mikael Carlsson, Daria Finocchiaro, Ulf Söderström, and Karl Walentin for their invaluable guidance and continuous support. For their useful feedback and comments, we also thank Itai Agur, Ragna Alstadheim, Darrel Duffie, Rod Garratt, Ricardo Lagos, Cyril Monnet, Dirk Niepelt (discussant), Anna Rogantini Picco (discussant), and Joshua Weiss (discussant), as well as conference and seminar participants at Uppsala University, New York University, the 2nd PhD Workshop in Money and Finance at Sveriges Riksbank, the 2nd Conference on the Economics of CBDC by the Bank of Canada and Sveriges Riksbank, the CEPR-ECB Conference on the Macroeconomic Implications of CBDCs, the 16th Nordic Summer Symposium in Macroeconomics, and the Economics of Payments XIII Conference at the Oesterreichische Nationalbank.

1 Introduction

Digital currencies have been around for a while, but their potential significance in the global economy has increased recently due to a growing demand for digital payment methods for retail purposes and the gradual decline of the use of cash for transactions in many economies [see, e.g., Khiaonarong and Humphrey (2022)]. Besides the private digital means of payment currently in circulation, many central banks have been investigating the possibility of launching a central bank digital currency (CBDC). CBDCs are central bank liabilities denominated in an existing unit of account that serve as a medium of exchange and a store of value.[1] Were CBDCs to be issued to retail customers (i.e., households), they would likely be a digital form of cash that share features with banknotes, as they are universally accessible but in a digital form.

Alongside an intense policy debate, a growing academic literature on the broader economic implications of CBDCs has emerged. A CBDC represents a novel payment alternative to cash and commercial bank deposits, with macroeconomic consequences that will affect both individuals and financial institutions. A primary concern for central banks when considering the issuance of a CBDC is the risk of the CBDC disintermediating the banking sector as households substitute the CBDC for bank deposits, potentially reducing bank profits and resulting in overall negative economic effects. This paper analyzes the implications of introducing a retail CBDC, particularly concerning its relationship with bank deposits.

The recent literature establishes an equivalence result between different payment systems. Brunnermeier and Niepelt (2019) consider a simplified scenario without reserves and resource cost of providing liquidity, while Niepelt (2022) includes a reserves layer and shows that introducing CBDC has no real effects on the economy if the private and the public sectors are equally efficient in operating payment systems. For this to happen, the central bank must refinance the bank at a lending interest rate that supports the bank's original portfolio position so that central bank funding exactly replaces the lost deposits for the bank.

Niepelt (2022) assumes central bank loans are extended against no collateral. However, the collateral requirement imposed by central banks when lending to commercial banks is potentially important for how introducing a CBDC may affect the banking sector and the real economy.[2] In practice, central banks lend to commercial banks (i.e., discount window lending) against collateral to support the liquidity and stability of the banking system. The liquidity provided by central banks helps financial institutions to manage their liquidity risks

[1] As defined by the Committee on Payments and Market Infrastructures of the Bank of International Settlements [Committee on Payments and Market Infrastructures - Markets Committee (CPMI-MC) (2018)].

[2] See, e.g., Burlon, Muñoz, and Smets (2024) and Williamson (2022).

efficiently. These loans are issued at an administered discount rate and must be collateralized to the satisfaction of the issuing central bank. In the euro area, banks can make use of the marginal lending facility, which enables banks to obtain overnight liquidity from the European Central Bank against sufficient eligible assets. In the United States, the Federal Reserve offers different types of discount window credit, which must be collateralized. The discount window mechanism has become increasingly important after the Global Financial Crisis. In this paper, we will revisit this equivalence result in the literature and explore its implication in terms of financial disintermediation by introducing a financial friction for central bank lending to banks (i.e., the collateral requirement).[3]

This paper addresses the potential risk of bank disintermediation following the introduction of a CBDC. To address this concern, we build on Niepelt (2022) and develop a model with a CBDC and bank deposits, adding a collateral requirement for central bank lending to banks. The framework is an extension of the model by Sidrauski (1967) that embeds a banking sector, bank deposits, government bonds, reserves, and a CBDC into the baseline real business cycle model. Households value goods, leisure, and the liquidity services that deposits and CBDC provide. Non-competitive banks invest in capital, reserves, and government bonds and fund themselves through either deposits or borrowing from the central bank. Firms produce using labor and physical capital. Finally, the consolidated government collects taxes, invests in capital, lends to banks against collateral, and issues CBDC and reserves.

After building our model, we use it to analyze two different perspectives: a static study comparing two equilibria with and without CBDC and a dynamic analysis of the economy's responses to an increase in the CBDC.

In the first part of the paper, we revisit the result in the literature regarding the equivalence between payment systems when introducing a collateral constraint for central bank lending to banks. Our findings reveal that the introduction of CBDC has no real effects on the economy as long as (i) CBDC and deposits are perfect substitutes, (ii) the resource cost per unit of effective real balances is the same for CBDC and deposits and (iii) the central bank offers a loan rate that renders the non-competitive banks indifferent to the introduction of the CBDC. Our equivalent central bank loan rate is lower than the one obtained in Niepelt (2022) because of the collateral requirement the bank must respect when borrowing from the central bank. In particular, when the collateral constraint becomes more restrictive, the central bank loan rate must be lower. However, different from the results in the literature, we find that, as households shift to CBDC, while the central bank can replace lost deposit funding for banks by lending to them, to access central bank loans banks must hold collaterals at the expense

[3]In Appendix C, we will revisit the equivalence result by considering different degrees of substitutability between CBDC and bank deposits (i.e., imperfect substitutability).

of extending credit to firms. In other words, the central bank insulates bank profits but not the bank's business model. It follows that, although the new policy has no real effects on the economy, it does not guarantee full neutrality as it affects the bank's business model.

In the second part of the paper, we explore the potential threat to bank intermediation should CBDC crowd out deposits. We depart from the equivalence analysis, where we compared two equilibria with and without CBDC, and use a dynamic model extension to study the economy's responses to an increase in the CBDC, where the central bank issues CBDC equal to a fraction of steady-state output. We find that the increased amount of CBDC in circulation boosts banks' capital. This result suggests that banks expand credit intermediation to firms. Interestingly, the CBDC does not lead to bank disintermediation or crowd out of deposits, but it expands bank activity.

Related literature. Our work contributes to the recent literature examining the impact of the introduction of CBDC on commercial banks. For instance, Chiu et al. (2023) develop a micro-founded general equilibrium model based on the framework of Lagos and Wright (2005) calibrated to the U.S. economy and find that a CBDC expands bank intermediation when the price of CBDC falls within a certain range while leading to disintermediation if its interest rate exceeds the upper limit of that range. In another study, using a dynamic banking model, Whited, Y. Wu, and Xiao (2023) assume that banks rely on deposits and wholesale funding and that the latter can potentially substitute deposit loss. Depending on whether the CBDC pays interest or not, the synergies between deposits and lending can attenuate the impact of CBDC. In another work, Keister and Sanches (2022) consider a competitive market and show that a deposit-like CBDC tends to crowd out bank deposits but, at the same time increases the aggregate stock of liquid assets in the economy, promoting more efficient levels of production and exchange and ultimately raising welfare. Similarly, Paul, Ulate, and J. C. Wu (2024) develop a dynamic stochastic general equilibrium (DSGE) model with monopolistic banks and find that introducing a CBDC enhances household liquidity and limits banks' market power over deposits, but it can also diminish bank lending and profitability, highlighting a critical welfare trade-off in CBDC design.

Specifically, our study is closer to the literature examining the CBDC introduction when banks borrow from the central bank subject to a collateral requirement. In recent work, Burlon, Muñoz, and Smets (2024) construct a quantitative euro area DSGE model, where banks must post government bonds as collateral to borrow from the central bank. They investigate the transmission channels of the issuance of CBDC to bank intermediation, finding a bank disintermediation effect with central bank financing replacing deposits, and government bonds displacing reserves and loans. Along similar lines, Assenmacher et al. (2021) use a DSGE model to investigate the macroeconomic effects of CBDC when the central bank administrates the CBDC rate and collateral and quantity requirements. Their findings indicate that a more ample supply of CBDC reduces bank deposits, while stricter collateral or quantitative constraints reduce welfare but can potentially contain bank disintermediation. The latter effect is particularly true when the elasticity of substitution between bank deposits and CBDC is low. Williamson (2022), on the other hand, explores the effects of the introduction of CBDC using a model of multiple means of payment. In his model, the CBDC is a more efficient payment instrument than cash, but it lengthens the central bank's balance sheet, creating collateral scarcity in the economy. Differently from these works, our study investigates the implications of CBDC issuance on bank intermediation using a real business cycle model that is closely connected to the baseline macroeconomic workhorse model, building on Niepelt (2022) and embedding a collateral requirement for central bank lending to banks. We evaluate the implications of CBDC issuance on bank intermediation with a dynamic model extension. Our findings suggest that introducing CBDC does not cause bank disintermediation but may foster an expansion of credit extension to firms.

Our study also contributes to the literature on the equivalence of payment systems. Existing work by Brunnermeier and Niepelt (2019) and Niepelt (2022) propose a compensation mechanism where the household's shift from deposits to CBDC can be offset by central bank lending to banks. However, these models abstract from the collateral constraint for central bank lending that is common in practice. Notably, Piazzesi and Schneider (2022) show that when banks are required to hold liquid assets to back their deposits and face asset management costs, the equivalence between alternative payment instruments breaks down, even if banks can be refinanced directly by the central bank. In light of this, we revisit the equivalence result by incorporating a collateral constraint for banks. We derive a new central bank lending rate that depends on the restrictiveness of the collateral requirement. Our findings reveal that the more restrictive the collateral constraint, the lower the loan rate the central bank must post. Additionally, we draw new insights on the mechanism behind the central bank compensation for banks. In our setting, to access central bank loans, banks must hold collaterals at the expense of extending credit to firms. Thus, while the CBDC introduction has no real effects on the economy, it does not guarantee full neutrality as it affects banks' business models.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 revisits and discusses the equilibrium analysis of the equivalence between operating payment systems. Section 4 characterizes the general equilibrium in which the household holds CBDC and deposits and discusses the dynamic effects of an increase in the CBDC. Section 5 concludes.

2 Model with CBDC and collateral-constrained banks

The model is based on Niepelt (2022) and describes an economy with a banking sector and CBDC in the absence of nominal rigidities. The CBDC and deposits provide direct utility. We depart from that framework by considering a collateral constraint for banks when borrowing from the central bank. There is a continuum of mass one of homogeneous infinitely-lived households who own a succession of two-period-lived banks and of one-period-lived firms. The consolidated government determines monetary and fiscal policy.

2.1 Households

The representative household wants to maximize the discounted felicity function $\mathscr{U}$, which is increasing, strictly concave and satisfies Inada conditions. Subject to its budget constraint, equation (1), the household takes wages, w_t; returns on asset i, R_t^i; profits, Π_t; and taxes, τ_t as given and solves

$$\max_{\{c_t, x_t, k_{t+1}^h, m_{t+1}, n_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mathscr{U}(c_t, x_t, z_{t+1})$$

$$\text{s.t.} \quad c_t + k_{t+1}^h + m_{t+1} + n_{t+1} + \tau_t = w_t(1 - x_t) + \Pi_t + k_t^h R_t^k + m_t R_t^m + n_t R_t^n, \quad (1)$$

$$k_{t+1}^h, m_{t+1}, n_{t+1} \geq 0,$$

where $\beta \in (0, 1)$ is the positive discount factor, c_t and x_t denote household consumption of the good and leisure at date t, respectively; k_{t+1}^h is capital at date $t + 1$; and $z_{t+1} = z(m_{t+1}, n_{t+1})$ are effective real balances carried from date t to $t + 1$. Effective real balances are a function of both CBDC, m_{t+1}, and bank deposits, n_{t+1}.[4] The household consumes, pays taxes, invests in capital, and has real balances, out of wage income, distributed profits and the gross return on the portfolio.

We focus on interior solutions for capital, CBDC, and deposits. To express the Euler equations for CBDC and deposits in a more compact form, we define the risk-free rate as

$$R_{t+1}^f = \frac{1}{\mathbb{E}_t \Lambda_{t+1}}, \quad (2)$$

[4] The household values liquidity, as suggested by the *money in the utility function* specification. In this setting, it only matters that the household demands liquidity services, not why they do.

where Λ_{t+1} is the household's stochastic discount factor:

$$\Lambda_{t+1} = \beta \frac{\mathscr{U}_c(c_{t+1}, x_{t+1}, z_{t+2})}{\mathscr{U}_c(c_t, x_t, z_{t+1})}. \tag{3}$$

Also, we define the liquidity premium, or interest spread, on asset i as

$$\chi^i_{t+1} = 1 - \frac{R^i_{t+1}}{R^f_{t+1}}, \qquad i \in \{m, n\}. \tag{4}$$

The spread on an asset i denotes the household's opportunity cost of holding said asset. A positive deposit spread, for instance, shows the interest return that the household forgoes by holding deposits. The household is willing to accept a lower return on deposits due to the liquidity service they provide. Assuming that the interest rates on CBDC and deposits are risk-free, we can summarize the first-order conditions as

$$x_t: \qquad \mathscr{U}_x(c_t, x_t, z_{t+1}) = \mathscr{U}_c(c_t, x_t, z_{t+1}) w_t, \tag{5}$$

$$k^h_{t+1}: \qquad 1 = \mathbb{E}_t R^k_{t+1} \Lambda_{t+1}, \tag{6}$$

$$m_{t+1}: \qquad \mathscr{U}_c(c_t, x_t, z_{t+1}) \chi^m_{t+1} = \mathscr{U}_z(c_t, x_t, z_{t+1}) z_m(m_{t+1}, n_{t+1}), \tag{7}$$

$$n_{t+1}: \qquad \mathscr{U}_c(c_t, x_t, z_{t+1}) \chi^n_{t+1} = \mathscr{U}_z(c_t, x_t, z_{t+1}) z_n(m_{t+1}, n_{t+1}). \tag{8}$$

The household's first-order conditions have standard interpretations. The leisure choice condition (5) equalizes the marginal benefit of leisure, $\mathscr{U}_x(c_t, x_t, z_{t+1})$, with its marginal cost in the form of reduced consumption due to less labor income, $\mathscr{U}_c(c_t, x_t, z_{t+1}) w_t$. The Euler equation for capital (6) dictates that the household saves in capital to the point where the marginal cost of saving in terms of consumption, $\mathscr{U}_c(c_t, x_t, z_{t+1})$, equals its expected discounted return, $\beta \mathbb{E}_t \mathscr{U}_c(c_{t+1}, x_{t+1}, z_{t+2}) R^k_{t+1}$. The first-order conditions for CBDC and deposits, equations (7) and (8) respectively, show that the household demands the liquid asset $i \in \{m, n\}$ to the point where its marginal benefit $\mathscr{U}_z(c_t, x_t, z_{t+1}) z_i(m_{t+1}, n_{t+1}) / \mathscr{U}_c(c_t, x_t, z_{t+1})$ equals its opportunity cost in terms of foregone interest, χ^i_{t+1}. Combining equations (7) and (8) we also see how the household trades off CBDC and deposits:

$$z_m(m_{t+1}, n_{t+1}) \chi^n_{t+1} = z_n(m_{t+1}, n_{t+1}) \chi^m_{t+1}. \tag{9}$$

Equation (9) shows that the household allocates between CBDC and deposits so that the marginal rate of substitution, $z_m(m_{t+1}, n_{t+1}) / z_n(m_{t+1}, n_{t+1})$, equals the relative price, $\chi^m_{t+1} / \chi^n_{t+1}$.

2.2 Banks

One of the often cited reasons in the literature for introducing a CBDC is bank market power [see, e.g., Andolfatto (2021), Garratt, Yu, and Zhu (2022)]. Specifically, banks offer lower deposit rates to extract rents, and households are willing to accept this markdown as they value the liquidity service provided by deposits. A CBDC could compete with bank deposits, reducing banks' market power.

Our set-up follows Niepelt (2022) and assumes that each bank is a monopsonist in its regional deposit market, such that the household in a region can only access the regional bank. A bank lives for two periods, and at date t issues deposits, n_{t+1}, and borrows from the central bank, l_{t+1}. It invests in reserves, r_{t+1}, government bonds, b_{t+1}, and capital, k^b_{t+1}.[5] Without loss of generality, we abstract from bank equity.

We follow Burlon, Muñoz, and Smets (2024) and assume that the bank is subject to a collateral requirement such that the loans they get from the central bank can not exceed a fraction θ_b of its government bond holdings. In this setting, government bonds are the only asset that can be pledged as collateral. For simplification, we abstract away from interbank loans with collateral. Holding government bonds gives liquidity benefits to the bank since they can use their holdings to obtain funding from the central bank. In other words, the bank is willing to forego a spread on the risk-free rate because of the collateral benefits of holding government bonds. This "convenience yield" of government bonds reflects the additional benefits the bank derives from holding these bonds beyond their financial yield. Therefore, government bonds are remunerated at a slightly lower rate than the risk-free rate.

The operating costs in the retail payment system, ν, are a decreasing function of the bank's reserve-to-deposit ratio, ζ_{t+1}. This is analogous to a binding minimum reserves requirement, as larger reserve holdings relative to deposits lower the bank's operating costs. We also allow ν to decrease with the stock of reserves and deposits of other banks, $\bar{\zeta}_{t+1}$, so as to capture positive externalities of reserve holdings.[6] To simplify the analysis, we make some assumptions which imply that in equilibrium $\zeta_{t+1} = \bar{\zeta}_{t+1}$, and reserves are strictly positive if and only if deposits are strictly positive: When a bank holds no deposits, its operating costs are null, and when all other banks have no deposits, the bank's operating costs are large but bounded. In this way, we rule out asymmetric equilibria in the bank's deposits and other banks' deposits. Otherwise, the operating cost function, $\nu(\zeta_{t+1}, \bar{\zeta}_{t+1})$, is strictly decreasing in both arguments, strictly convex, and satisfies $\nu_{\zeta\bar{\zeta}} = 0$ and $\nu_{\zeta\zeta} \geq \nu_{\bar{\zeta}\bar{\zeta}}$, as well as $\lim_{\zeta_{t+1}\to 0} \nu_\zeta = \infty$.

[5] Bank's capital is defined as $k^b_{t+1} = n_{t+1} + l_{t+1} - r_{t+1} - b_{t+1}$. Alternatively, the bank can invest in loans to firms that eventually fund physical capital accumulation.

[6] Niepelt (2022) uses a cost function in the form $\nu + \omega_t(\zeta_{t+1}, \bar{\zeta}_{t+1})$, where ν is the resource cost per unit of deposit funding, and ω represents the bank's resource costs of liquidity substitution.

The bank chooses the quantity of deposits and central bank loans subject to the deposit funding schedule of the household.[7] Since the bank acts as a monopsonist in its regional deposit market, it takes the deposit funding schedule (rather than the deposit and the central bank loan rates) as given. The program of the bank at date t reads

$$
\begin{aligned}
& \max_{n_{t+1}, l_{t+1}, r_{t+1}, b_{t+1}} \Pi_{1,t}^{b} + \mathbb{E}_t \left[\Lambda_{t+1} \Pi_{2,t+1}^{b} \right] \\
\text{s.t.} \quad & \Pi_{1,t}^{b} = -n_{t+1} \nu(\zeta_{t+1}, \bar{\zeta}_{t+1}), && (10) \\
& \Pi_{2,t+1}^{b} = (n_{t+1} + l_{t+1} - r_{t+1} - b_{t+1}) R_{t+1}^{k} \\
& \qquad + r_{t+1} R_{t+1}^{r} + b_{t+1} R_{t+1}^{b} - n_{t+1} R_{t+1}^{n} - l_{t+1} R_{t+1}^{l}, && (11) \\
& l_{t+1} \leq \theta_b \frac{b_{t+1}}{R_{t+1}^{l}}, && (12) \\
& R_{t+1}^{n}, R_{t+1}^{l} \text{ perceived endogenous}, \\
& n_{t+1}, l_{t+1}, b_{t+1} \geq 0,
\end{aligned}
$$

where

$$
\zeta_{t+1} \equiv \frac{r_{t+1}}{n_{t+1}}, \qquad \bar{\zeta}_{t+1} \equiv \frac{\bar{r}_{t+1}}{\bar{n}_{t+1}},
$$

and $\Pi_{1,t}^{b}$, $\Pi_{2,t+1}^{b}$ denote the cash flow generated in the first and second periods of the bank's operations, respectively.

We focus on interior solutions for deposits, loans, and government bonds, and we make use of the risk-free rate and the household's first-order condition for capital, equations (2) and (6), respectively. Also, we define the elasticity of the asset i with respect to the rate of return on i as

$$
\eta_{i,t+1} = \frac{\partial i_{t+1}}{\partial R_{t+1}^{i}} \frac{R_{t+1}^{i}}{i_{t+1}}, \qquad i \in \{n, l\},
$$

and the liquidity premia on central bank loans, reserves, and government bonds as in equation (4). Let γ_t denote the Lagrange multiplier associated with the collateral constraint. The

[7] In the model, the central bank's loan funding schedule replicates the household's deposit funding schedule. This assumption plays a crucial role in the context of the equivalence analysis.

collateral constraint is binding in equilibrium, such that[8]

$$\gamma_t > 0, \qquad l_{t+1} = \theta_b \frac{b_{t+1}}{R^l_{t+1}}.$$

We can write the bank's optimality conditions as

$$n_{t+1}: \qquad \chi^n_{t+1} - \Big(\nu(\zeta_{t+1}, \bar{\zeta}_{t+1}) - \nu_\zeta(\zeta_{t+1}, \bar{\zeta}_{t+1})\zeta_{t+1}\Big) = \frac{1}{\eta_{n,t+1}} \frac{R^n_{t+1}}{R^f_{t+1}}, \tag{13}$$

$$r_{t+1}: \qquad - \nu_\zeta(\zeta_{t+1}, \bar{\zeta}_{t+1}) = \chi^r_{t+1}, \tag{14}$$

$$l_{t+1}: \qquad \chi^l_{t+1} - \gamma_t \left(1 + \frac{1}{\eta_{l,t+1}}\right) = \frac{1}{\eta_{l,t+1}} \frac{R^l_{t+1}}{R^f_{t+1}}, \tag{15}$$

$$b_{t+1}: \qquad \gamma_t \frac{\theta_b}{R^l_{t+1}} = \chi^b_{t+1}, \tag{16}$$

where the spreads on bonds and central bank loans, χ^b_{t+1} and χ^l_{t+1}, are defined in the same way as the spreads on deposits and CBDC given by expression (4).

We first comment on the liability side of the bank's balance sheet, starting with deposits. The left-hand side of the equation (13) represents the marginal profit from issuing deposits, which is given by the difference between the bank's gain from the positive deposit liquidity premium and the marginal cost associated with increased deposit issuance. The right-hand side equals the marginal cost of inframarginal deposits, as higher deposit issuance is associated with an increased interest rate on deposits. Similarly, the condition for central bank loans, equation (15), states that the sum of the bank's marginal benefits of taking on more central bank loans and the gain coming from the positive loan liquidity premium should be equal to the marginal cost associated with central bank loans. In fact, higher loan holdings are associated with an increase in the interest rate on the central bank loans.

Turning now to the asset side of the bank's balance sheet, equation (14) equalizes the marginal benefit of reserves in the form of reduced operating costs with the bank's opportunity cost of reserves. Looking at equation (16), the optimal choice of government bonds is when the bank's marginal costs of bond holdings are equal to the loss coming from the bank's lower return with a positive spread on government bonds.

[8]See Appendix A for the conditions under which the collateral constraint binds. The intuition is that, with a non-binding collateral constraint, in equilibrium $\gamma_t = 0$ and, from the collateral constraint condition (12), $0 \leq \theta_b \frac{b_{t+1}}{R^l_{t+1}} - l_{t+1}$. However, from the government bonds optimality condition (not shown here), this violates the condition that $R^b_{t+1} < R^f_{t+1}$, so the collateral constraint must bind in equilibrium.

Combining equations (13) and (14) yield

$$\chi^n_{t+1} - \left[\nu(\zeta_{t+1}, \bar{\zeta}_{t+1}) + \chi^r_{t+1}\zeta_{t+1}\right] = \frac{1}{\eta_{n,t+1}}\frac{R^n_{t+1}}{R^f_{t+1}}. \tag{13a}$$

This implies that the bank's net benefit of issuing more deposits must equal the inframarginal cost of deposits. Combining equations (15) and (16) results in the relation

$$\chi^l_{t+1} - \chi^b_{t+1}\frac{R^l_{t+1}}{\theta_b}\left(1 + \frac{1}{\eta_{l,t+1}}\right) = \frac{1}{\eta_{l,t+1}}\frac{R^l_{t+1}}{R^f_{t+1}}. \tag{15a}$$

The marginal cost of taking on more central bank loans must equal the bank's net benefit of taking on more loans. This is given by the difference between the liquidity benefits given by the central bank loans and the marginal cost associated with the collateral constraint.

2.3 Firms

Neoclassical firms live for one period and rent capital, k_t, and labor, ℓ_t, to produce the output good to maximize the profit, Π^f_t. The representative firm takes wages, w_t; the rental rate of capital, $R^k_t + \delta - 1$; and the good price as given and solves

$$\max_{k_t,\ell_t} \Pi^f_t$$
$$\text{s.t.} \qquad \Pi^f_t = f(k_t, \ell_t) - k_t(R^k_t + \delta - 1) - w_t\ell_t,$$

where f is the neoclassical production function. The first-order conditions read

$$k_t: \qquad f_k(k_t, \ell_t) = R^k_t + \delta - 1, \tag{17}$$
$$\ell_t: \qquad f_l(k_t, \ell_t) = w_t. \tag{18}$$

2.4 Consolidated government

The consolidated government collects taxes, lends to the bank against collateral, invests in capital, k^g_{t+1}, and issues CBDC and reserves. The government budget constraint reads

$$\begin{aligned} k^g_{t+1} + l_{t+1} - b_{t+1} - m_{t+1} - r_{t+1} =& k^g_t R^k_t + l_t R^l_t - b_t R^b_t - m_t R^m_t - r_t R^r_t \\ &+ \tau_t - m_{t+1}\mu - r_{t+1}\rho, \end{aligned} \tag{19}$$

where μ and ρ are the unit resource costs of issuing (and managing) CBDC and reserves payments, respectively.

2.5 Market clearing

Market clearing in the labor market requires that the firm's labor demand equals the household's labor supply:

$$\ell_t = 1 - x_t. \tag{20}$$

Market clearing for capital requires that the firm's demand for capital equals capital holdings of the household, the bank, and the government:

$$k_t = k_t^h + (n_t + l_t - r_t - b_t) + k_t^g. \tag{21}$$

Profits distributed to the household must equal the sum of the bank and firm profits:

$$\Pi_t = \Pi_{1,t}^b + \Pi_{2,t}^b + \Pi_t^f. \tag{22}$$

By Walras' law, market clearing on labor and capital markets and the budget constraints of the household, bank, firm, and consolidated government imply market clearing on the goods market.

To derive the aggregate resource constraint for the economy, we plug equation (22) into the household's budget constraint, equation (1), and we impose market clearing conditions (20) and (21). Then, in combination with the government's budget constraint, equation (19), the resulting expression is the aggregate resource constraint:

$$k_{t+1} = f(k_t, 1 - x_t) + k_t(1 - \delta) - c_t - \left(m_{t+1}\mu + n_{t+1}\left(\nu(\zeta_{t+1}, \bar{\zeta}_{t+1}) + \zeta_{t+1}\rho\right)\right). \tag{23}$$

3 Revisiting the equivalence of payment instruments

In this section, we use our model in a static study comparing two equilibria with and without CBDC and revisiting the result in the literature regarding the equivalence of payment instruments. Existing works by Brunnermeier and Niepelt (2019) and Niepelt (2022) suggest a compensation mechanism where the households' shift from deposits to CBDC can be offset by central bank lending to banks. We build upon these insights by incorporating a collateral constraint for central bank lending to banks, as specified in equation (12) from the bank's problem. We assume perfect substitutability of CBDC and deposits in the household's real balances, such that effective real balances are a weighted sum of the two instruments:

$$z_{t+1} = \lambda m_{t+1} + n_{t+1}, \tag{24}$$

where $\lambda \geq 0$ represents the benefits of CBDC relative to deposits. Besides reflecting the liquidity benefits or ease of use of CBDC, λ can also capture factors such as privacy considerations and payment security. The assumption that an interest-bearing CBDC and deposits are close substitutes is common in the literature [see, e.g., Andolfatto (2021) and Whited, Y. Wu, and Xiao (2023)] and is consistent with the CBDC experiments central banks are currently running.[9]

The following proposition is formally proved in Appendix B.

Proposition 1. *Consider a policy that implements an equilibrium with deposits, reserves, central bank loans, and government bonds, where the latter are used as banks' collateral to obtain central bank loans. The central bank introduces a new payment instrument, CBDC, which is a perfect substitute for deposits. There exists another policy and equilibrium with less deposits and reserves, a positive amount of CBDC, central bank loans, and government bonds, a different ownership structure of capital, additional taxes on the household, and otherwise the same equilibrium allocation and price system. In this equilibrium, the public and private sectors are equally efficient in providing liquidity to the household:*

$$\frac{\mu}{\lambda} = \nu(\zeta_{t+1}, \zeta_{t+1}) + \zeta_{t+1}\rho, \tag{25}$$

and the central bank lends to the bank at a loan rate equal to

$$R^l_{t+1} = \frac{R^n_{t+1} + \nu(\zeta_{t+1}, \zeta_{t+1})R^f_{t+1} - \zeta_{t+1}R^r_{t+1}}{(1-\zeta_{t+1})\left(1 + \frac{R^k_{t+1} - R^b_{t+1}}{\theta_b}\right)}. \tag{26}$$

■

Condition (25) stipulates that the resource cost the government pays to provide one unit of real balances through CBDC equalizes the corresponding cost for deposits, which is the sum of the operating cost incurred by the bank, and the resource cost associated with reserves incurred by the government. This condition is realistic since one of the options central banks are considering for issuing a CBDC to the public involves using the existing commercial banks'

[9] See Appendix C for the equivalence study in the case of CBDC and deposits as imperfect substitutes.

deposit distributing systems.[10] A central bank's loan rate equal to the value in equation (26) ensures that (i) the market values of taxes on the household and of changes in bank profits are zero; (ii) the government budget constraint is unaffected by the new policy; and (iii) the bank chooses loans to make up for the reduction in funding from the household.

The loan rate we derive is lower than the one in Niepelt (2022) due to the extra term $\left(1 + \frac{R^k_{t+1} - R^b_{t+1}}{\theta_b}\right) > 0$ in equation (26).[11] It follows that when the bank is subject to a collateral constraint for central bank lending, the central bank must post a lower rate than in a no-collateral constraint scenario. The intuition is that when the bank is not collateral-constrained, it can borrow as much as it wants from the central bank. With a collateral requirement, the central bank needs to offer a lower lending rate to incentivize the bank to borrow the same quantity as in the absence of the constraint, such that it remains indifferent to the introduction of the CBDC.[12] Note that, since Niepelt (2022) does not account for the central bank's collateral requirement, it is like considering $\theta_b \to +\infty$. Calling the central bank loan rate obtained in Niepelt (2022) as $\tilde{R}^l_{t+1}$, we can conclude that $\lim_{\theta_b \to +\infty} R^l_{t+1} = \tilde{R}^l_{t+1}$.

The central bank loan rate we derive in equation (26) depends on how restrictive is the collateral constraint: the tighter the constraint is (the lower θ_b), the lower the lending rate the central bank needs to offer (the lower R^l_{t+1}). The central bank loan rate is higher than 1 for reasonable values of θ_b and is at its highest when all bonds held by the bank can be pledged, i.e., $\theta_b = 1$.

A final remark is worth noting. A central bank's loan rate value as in equation (26) insulates the bank's profits. However, when the bank is collateral-constrained, to access central bank loans, banks must hold government bonds as collateral at the expense of extending credit to firms, and the central bank assumes part of the credit-extension role. Figure 1 reports the bank's balance sheet breakdown comparing the two equilibria before and after the introduction of the CBDC.[13]

The following corollary follows from Proposition 1:

Corollary 1. *The central bank loan rate as in equation (26) insulates bank profits but not the bank's business model. Although the new policy has no real effects on the economy, it does*

[10] See, e.g., Kosse and Mattei (2023) for the results of the 2022 BIS survey on central bank digital currencies and crypto.

[11] From the household's problem, we know that $R^k_{t+1} \leq R^f_{t+1}$, assuming that the rate of return on capital is not risky, we can approximate $R^k_{t+1} \simeq R^f_{t+1}$. We also know that for the bank there is a collateral premium associated with holding government bonds, thus $R^b_{t+1} < R^f_{t+1}$. It follows that the extra term is positive.

[12] We abstract from any social cost associated with central bank lending to banks.

[13] In the banks' balance sheet before the CBDC introduction, government bonds and central bank loans are normalized to zero.

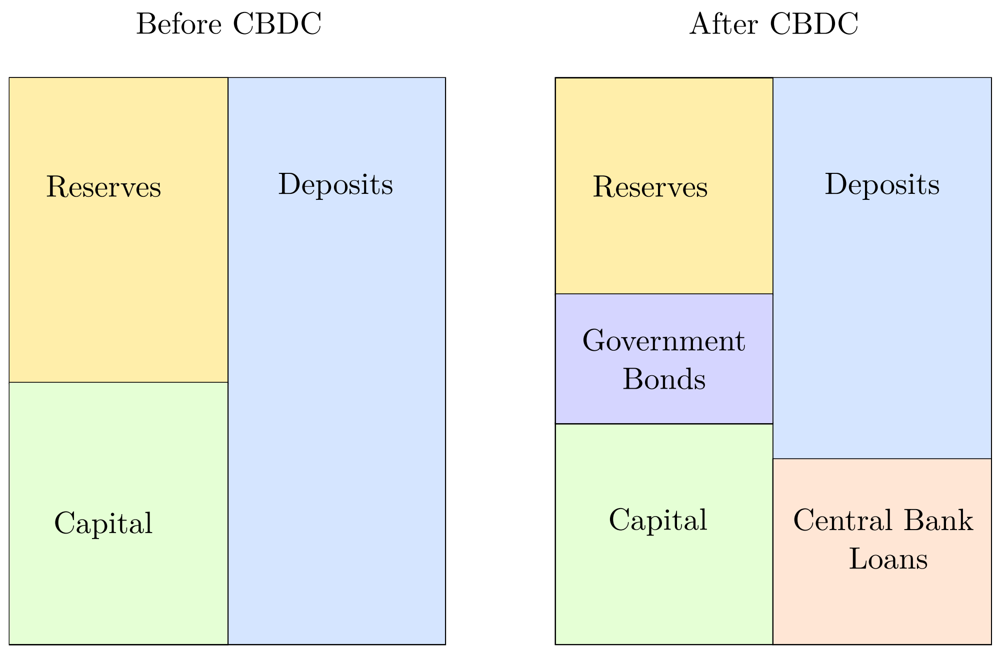

Figure 1: Bank's balance sheets before and after the CBDC introduction

not guarantee full neutrality as it changes the bank's business model.

■

3.1 Discussion

The equivalence result in the literature is akin to a Modigliani-Miller benchmark, suggesting that the introduction of CBDC will not affect the real economy through reduced credit provision, as long as the central bank offsets any disintermediation of banks. The idea is that when households shift from deposits to CBDC, the central bank can lend to banks to replace the lost deposits, maintaining the overall credit supply in the economy.

We extend this framework by introducing financial friction in central bank lending to banks, adding realism to the model. In our setting, banks are required to post government bonds as collateral to borrow from the central bank. Due to this collateral constraint, as households partially substitute deposits with CBDC, banks replace their lost funds with central bank loans, but to do so, they must redirect some of their capital investments and reserves towards purchasing government bonds, as illustrated in Figure 1. As a result, the government compensates for the shortfall in banks' capital investments, ensuring that CBDC introduction does not impact aggregate capital or real production. However, this adjustment reshapes bank balance sheets, ultimately leading to a change in their business model.

The scope of this analysis is to contribute to the theoretical rationale for why the equivalence result may not fully hold. Although the central bank can prevent real effects on the economy, the transformation of bank balance sheets and the change in their business model suggest that the impact of CBDC introduction may not be entirely neutral.

At the same time, as banks reduce their capital investments to meet collateral requirements, the central bank takes on some of the credit extension role, which is undesirable. In practice, central banks are cautious about placing themselves in a position where they need to re-intermediate lost bank funding. For this reason, many central banks are designing CBDCs with limited disintermediation in mind, for instance by considering caps on CBDC wallets.

4 Dynamic effects of introducing CBDC

In Section 3, we revisited the result regarding the equivalence of payment instruments by considering a collateral constraint for central bank lending to banks. We compared two equilibria, one with no CBDC and one when the central bank introduces a CBDC that competes with deposits. Next, we extend our model to study the dynamic effects of introducing CBDC, more specifically, the potential threat to bank intermediation should the CBDC crowd out deposits.

4.1 Functional forms and equilibrium conditions

The functional form for real balances is represented by equation (24). We assume that the household has utility function of the form

$$\mathscr{U}(c_t, x_t, z_{t+1}) = \frac{\left((1-\iota)c_t^{1-\psi} + \iota z_{t+1}^{1-\psi}\right)^{\frac{1-\sigma}{1-\psi}}}{1-\sigma} x_t^{\upsilon}, \tag{27}$$

where $\iota > 0$ is the utility weight of liquidity; $\sigma > 0$ is the inverse intertemporal elasticity of substitution between bundles of consumption and real balances across times; $\psi > 0$ is the inverse intratemporal elasticity of substitution between consumption and real balances; and υ is the exponent of the power function for leisure. The bank's operating cost function has the following form:

$$\nu(\zeta_{t+1}, \bar{\zeta}_{t+1}) = \phi_1 \zeta_{t+1}^{1-\varphi} + \phi_2 \bar{\zeta}_{t+1}^{1-\varphi}, \tag{28}$$

where $\phi_1, \phi_2 \geq 0$ are the relative weights assigned to the bank's reserves-to-deposit ratio and to the other bank's ratio; and $\varphi > 1$. Lastly, the firm has the standard Cobb-Douglas production function:

$$f(k_t,\ell_t)=k_t^{\alpha}\ell_t^{1-\alpha}, \tag{29}$$

where k_t and ℓ_t are the firm's demand for capital and labor, respectively, and α is the capital share of output.

Given the functional form assumptions, we characterize the general equilibrium. First, knowing the household's utility functional form, we can rewrite the stochastic discount factor, equation (3) as

$$\Lambda_{t+1}=\beta\frac{c_{t+1}^{-\sigma}x_{t+1}^{\upsilon}\Omega_{t+1}^{c}}{c_t^{-\sigma}x_t^{\upsilon}\Omega_t^{c}},$$

so that the risk-free rate, equation (2), is given by

$$R_{t+1}^{f}=\frac{1}{\mathbb{E}_t\left[\Lambda_{t+1}\right]}.$$

The household's capital Euler equation (6) and leisure choice condition (5), and the aggregate resource constraint (23) become, respectively,

$$1=\mathbb{E}_t\left[\Lambda_{t+1}R_{t+1}^{k}\right], \tag{30}$$

$$\frac{c_t^{1-\sigma}}{1-\sigma}\upsilon x_t^{\upsilon-1}\Omega_t^{x}=w_t c_t^{-\sigma}x_t^{\upsilon}\Omega_t^{c}, \tag{31}$$

$$k_{t+1}=k_t^{\alpha}(1-x_t)^{1-\alpha}+k_t(1-\delta)-c_t-\left(m_{t+1}\mu+n_{t+1}\left(\nu(\zeta_{t+1},\zeta_{t+1})+\zeta_{t+1}\rho\right)\right), \tag{32}$$

where

$$\Omega_t^{c}=(1-\iota)^{\frac{1-\sigma}{1-\psi}}\left(1+\left(\frac{\iota}{1-\iota}\right)^{\frac{1}{\psi}}\left(\chi_{t+1}^{n}\right)^{1-\frac{1}{\psi}}\right)^{\frac{\psi-\sigma}{1-\psi}},$$

$$\Omega_t^{x}=(1-\iota)^{\frac{1-\sigma}{1-\psi}}\left(1+\left(\frac{\iota}{1-\iota}\right)^{\frac{1}{\psi}}\left(\chi_{t+1}^{n}\right)^{1-\frac{1}{\psi}}\right)^{\frac{1-\sigma}{1-\psi}}.$$

These equilibrium conditions closely parallel those of a standard real business cycle model. Unlike the standard model, however, the auxiliary variables Ω_t^c and Ω_t^x, summarize the impact of the household's preference for liquidity on consumption/savings and leisure choices. Moreover, the term $m_{t+1}\mu+n_{t+1}\left(\nu(\zeta_{t+1},\zeta_{t+1})+\zeta_{t+1}\rho\right)$ in the resource constraint shows the societal costs of providing liquidity to the household incurred by the government and banks.

We combine the household's first-order condition for deposits, equation (8), with the expression for real balances, equation (24), to derive the deposit demand:

$$n_{t+1} = c_t \left(\frac{\iota}{1-\iota} \frac{1}{\chi^n_{t+1}} \right)^{\frac{1}{\psi}} - \lambda m_{t+1}, \tag{33}$$

where we combine the first-order conditions for deposits and reserves from the bank's problem, (13) and (14), to derive the expressions for the equilibrium deposit spread, χ^n_{t+1}:

$$\chi^n_{t+1} = \frac{(\phi_1 \varphi + \phi_2)\zeta_{t+1}^{1-\varphi}}{1 - \psi \frac{n_{t+1}}{z_{t+1}}}, \tag{34}$$

and the bank's optimal reserves-to-deposits ratio, ζ_{t+1}, depends on the spread on reserves, χ^r_{t+1},

$$\zeta_{t+1} = \left(\frac{\chi^r_{t+1}}{\phi_1(\varphi - 1)} \right)^{-\frac{1}{\varphi}}. \tag{35}$$

Given the real balances functional form assumption, equation (24), from the household's problem equation (9) we derive the CBDC spread as

$$\chi^m_{t+1} = \lambda \chi^n_{t+1}. \tag{36}$$

Note that the spread on reserves is derived from equation (4).

From the binding collateral constraint, equation (12), we derive the bank's demand for government bonds as

$$b_{t+1} = \frac{l_{t+1} R^l_{t+1}}{\theta_b}. \tag{37}$$

Combining the bank's optimality conditions for central bank loans and government bonds, equations (15) and (16), we derive the bank's demand for central bank loans. We restrict our attention to the case where this demand is non-negative, i.e.,[14]

$$l_{t+1} = \begin{cases} \left(\chi^l_{t+1} - \chi^b_{t+1} \frac{R^l_{t+1}}{\theta_b} \right) \left(\frac{\theta_b}{\chi^b_{t+1} R^f_{t+1} + \theta_b} \right) \frac{z_{t+1}}{\psi \chi^n_{t+1}} & \text{if } \chi^l_{t+1} - \chi^b_{t+1} \frac{R^l_{t+1}}{\theta_b} \geq 0 \\ 0 & \text{if } \chi^l_{t+1} - \chi^b_{t+1} \frac{R^l_{t+1}}{\theta_b} < 0 \end{cases}. \tag{38}$$

[14]To derive equation (38), we assume that the central bank's loan supply schedule replicates the household's deposit demand schedule, i.e.,

$$\frac{\partial l_{t+1}}{\partial R^l_{t+1}} = \frac{\partial n_{t+1}}{\partial R^n_{t+1}}.$$

Finally, from the firm's optimality conditions, equations (17) and (18), we derive the return on capital and the real wage:

$$R_t^k = 1 - \delta + \alpha \left(\frac{k_t}{1 - x_t} \right)^{\alpha - 1}, \tag{39}$$

$$w_t = (1 - \alpha) \left(\frac{k_t}{1 - x_t} \right)^{\alpha}. \tag{40}$$

4.2 Shock

After characterizing the general equilibrium, we aim to investigate the potential threat to bank intermediation should the CBDC crowd out deposits. We address this concern by studying the economy's responses to an increase in CBDC. We follow Burlon, Muñoz, and Smets (2024) and assume that the central bank issues CBDC according to a policy rule which stipulates that CBDC is equal to a fraction of steady-state output, y:

$$m_{t+1} = \theta_t^m y. \tag{41}$$

In the baseline, the CBDC share, θ_t^m, follows an AR(1) process of the form

$$\theta_t^m = \rho^\theta \theta_{t-1}^m + e_t, \tag{42}$$

where ρ^θ is the persistence parameter and e_t is the exogenous one-time shock.

4.3 Calibration

The model is quarterly, and we calibrate it to the U.S. economy. We use variables without subscripts to denote their steady-state values. Table 1 summarizes the baseline calibration.

4.3.1 Households

The household's discount factor, β, is set to the standard value of 0.99. We assume that the household perceives CBDC and deposits as equally useful, i.e., $\lambda = 1$. We set the inverse intertemporal elasticity of substitution, σ, to 0.5. The leisure function coefficient, υ, is set to 0.858 to match a steady-state labor supply of 1/3. We assume that consumption and liquidity services are complements. Therefore, the inverse intratemporal elasticity of substitution between the two, ψ, is set higher than σ and equal to 0.55. We calibrate the utility weight of liquidity, ι to 0.025 to match the ratio of liquidity to output of 1.04, in line with Kaplan, Moll, and Violante (2018) and Bayer et al. (2019).

4.3.2 Banks and firms

Following Niepelt (2024), we set the bank operating cost parameter φ to 1.5034. We assume the internal and external costs of lack of reserves are identical, i.e. $\phi_2 = \phi_1 = \phi$. We then set ϕ to 0.002 to achieve a reserves-to-deposits ratio of 0.1945, as estimated by Niepelt (2024). The production sector is standard. The capital share of output, α, and the rate of capital depreciation, δ, are set to 1/3 and 0.025, respectively.

4.3.3 Government

We follow Niepelt (2024) and set the government's marginal cost of providing reserves, ρ, to $3.146 \cdot 10^{-4}$. In line with our discussion within the equivalence analysis in Section 3, we assume that the government is equally efficient in providing liquidity to the household as the banking sector. Thus, we set the government's cost of issuing CBDC, $\mu = 0.002$, in line with condition (25). Similarly, the interest rate on central bank loans, $R^l = 0.985$, is set to reflect the central bank loan rate we derived in condition (26). For simplicity, we assume that central bank reserves and government bonds are non-interest bearing (in real terms). The persistence in the supply of CBDC, ρ^θ, is set to the standard value of 0.9. Lastly, we set the haircut on government bonds to 0.5% which implies a collateral haircut $\theta_b = 0.995$.

4.4 Impulse responses

In this section, we keep the interest rates on reserves, central bank loans, and government bonds constant at their steady-state levels. The impulse responses are reported as percentage or basis point deviations from steady-state. Note that CBDC deviations are reported in absolute values because in the steady state there is no CBDC.

Figure 2 illustrates the impulse responses to a temporary increase in the CBDC share of steady-state output, θ_t^m, from zero to 5%, where θ_t^m follows equation (42). This initial increase in the CBDC share aligns with ranges considered in the recent literature. For instance, Abad, Nuño, and Thomas (2023) analyze the economy transitional dynamics in scenarios with steady-state take-up of CBDC equal to 4% and 7% of GDP.

As the share of CBDC increases, there is now a positive amount of CBDC in circulation. The equilibrium deposit spread, pinned down by equation (34), depends on the reserve spread, which determines the marginal cost of issuing deposits, and the share of deposits in total

Parameter	Value	Source/Motivation	Description
Households			
β	0.99	Standard	Discount factor
λ	1	Assumption	Relative CBDC benefit
σ	0.5	Assumption	Risk aversion
υ	0.858	$\ell = 1/3$	Leisure function coefficient
ψ	0.55	$\psi > \sigma$	Inv. elasticity of sub. c_t and z_{t+1}
ι	0.025	$z/y = 1.04$	Liquidity utility weight
Banks			
φ	1.5034	Niepelt (2024)	Operating cost
ϕ_1	0.002	$\zeta = 0.1945$	Operating cost
ϕ_2	0.002	$\phi_1 = \phi_2$	Operating cost
Firms			
α	1/3	Standard	Capital share of output
δ	0.025	Standard	Capital depreciation rate
Government			
ρ	$3.146 \cdot 10^{-4}$	Niepelt (2024)	Reserves cost
μ	0.002	Condition (25)	CBDC cost
R^l	0.985	Condition (26)	Central bank loans returns
R^r	1.0	Assumption	Reserves return
R^b	1.0	Assumption	Government bonds return
ρ^θ	0.9	Standard	CBDC supply persistence
θ_b	0.995	Haircut on bonds 0.5%	Collateral haircut

Table 1: Model Parameters

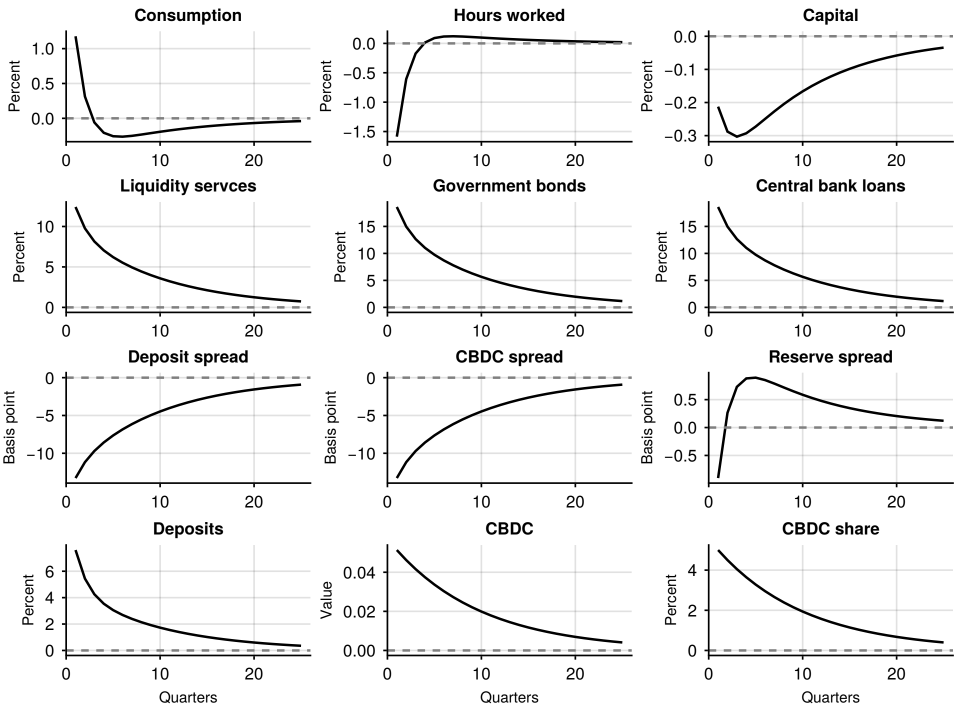

Figure 2: Impulse responses to a temporary increase in CBDC share from zero to 5%

liquidity services, which affects the markup the bank can charge on its prices over the marginal cost. The reserve spread falls only marginally, leaving the bank's marginal cost of issuing deposits largely unchanged. However, as CBDC enters circulation, deposits' share of liquidity falls. This reduces the bank's market power, represented by the term $1/(1-\psi\frac{n_{t+1}}{z_{t+1}})$ in equation (34), leading to a reduction in the deposit spread.

Since deposits and CBDC are perfect substitutes, the government ensures that both assets circulate by setting the return on CBDC such that equation (36) holds. The government ensures that the opportunity costs of holding deposits and CBDC are equalized, adjusting for the relative benefit of CBDC. As a result, the CBDC spread decreases by the same magnitude as the deposit spread.

Equation (33) shows that the household's demand for deposits decreases in both the deposit spread and the amount of CBDC in circulation. In our exercise, deposit demand increases because the effect of the declining deposit spread is stronger than the increase in CBDC. The increase in deposits is accompanied by an increase in the bank' demand for central bank loans, enabling the bank to expand its balance sheet. According to equation (38), this expansion is primarily driven by the greater availability of total liquid assets and the reduced deposit spread. To secure central bank loans, the bank must post collateral, leading to an increase in government bond holdings.

More aggregate liquidity also increases the current marginal utility of consumption. In other words, the opportunity cost of saving increases, incentivizing the household to save less and consume more. At the same time, the household's marginal benefit of leisure is now higher than the marginal cost, inducing them to decrease labor supply.

The aggregate capital in the economy is the sum of capital held by households, banks, and the government. Before the shock, the capital was mainly held by households. After the shock, banks and the government hold more, and aggregate capital drops because of the decrease in household capital (see Figure 3 in Appendix D for the aggregate capital breakdown by components). The expansion in the bank's capital follows from the liability side increasing less than the change in the sum of reserves and government bonds. Interestingly, the intuition behind the bank's capital increase is that the bank expands credit intermediation to firms. In other words, introducing CBDC not only does not cause bank disintermediation but expands bank activity.

In practice, the initial introduction of CBDC is likely to be gradual and permanent. To analyze the effects of such a change, we follow Chen et al. (2023) and model a near-permanent, gradual increase in the CBDC share using an AR(2) process.[15] Figure 4 in Appendix D shows the impulse responses of a gradual increase in the CBDC share that reaches 5% after approximately 20 quarters. Crucially, the responses of the banking sector remain qualitatively the same as those observed under the standard temporary shock illustrated in Figure 2. As CBDC circulation gradually rises, the deposit spread falls, and the quantity of deposits increases. The bank expands its balance sheet by borrowing more from the central bank and increases its bond holdings to meet the growing need for collateral.

Lastly, although the equivalence result shows that the introduction of CBDC does not change the allocation, the existence of CBDC might affect the transmission of shocks. Figure 5 in Appendix D shows the impulse responses to a temporary increase in the CBDC share where in the steady-state there already is CBDC equivalent to 5% of output. As the CBDC share increases from 5% to 7%, the qualitative characteristics of the responses remain the same as in the baseline.

4.5 Robustness checks

Uncertainty regarding the household's perception of CBDC usefulness would be important for any practical implementation of CBDC. Therefore, we first test the robustness of our results by changing the relative benefit of CBDC. Secondly, given the central role of the pledgeability

[15] The AR(2) process is defined as $\Delta\theta^m_t = 0.76 * \Delta\theta^m_{t-1} + e$

of bonds for our equivalence results, we test for different magnitudes for the haircuts on bonds.

First, we change the relative benefit of CBDC, λ. Figures 6 and 7 in the Appendix D.1 show the impulse responses to a temporary increase in CBDC share, θ_t^m, from zero to 5%, when λ is 0.5 and 1.5, respectively. Comparing these responses to the main specification in Figure 2, we see that the results remain qualitatively the same. The magnitudes of the responses are smaller when the relative benefit of CBDC is smaller, i.e. when $\lambda = 0.5$. This is because each unit of CBDC contributes less to total liquidity as CBDC comes into circulation, which in turn means the share of deposits in total liquidity, n_{t+1}/z_{t+1}, is at a higher level immediately after the shock than in the baseline. Intuitively, this means that for the same increase in CBDC, its competitive impact on the bank is smaller. Then, according to equation (34), the bank does not have to lower its deposit spread as much as in the baseline. The opposite is true when λ is higher.

Next, Figures 8 and 9 show the Impulse responses to a temporary increase in CBDC share when bond haircuts are at 0.1% ($\theta_b = 0.999$) and 1.5% ($\theta_b = 0.985$), respectively. We see that the changes in bond pledgeability do not alter the responses in shape or magnitude.

5 Conclusion

This paper investigates the potential risk to bank intermediation after introducing a CBDC competing with commercial bank deposits as the household's source of liquidity. We model CBDC and banks subject to a collateral requirement and analyze the static and dynamic effects of the CBDC.

We revisit the results on the equivalence of payment instruments when introducing a collateral constraint for central bank lending to banks. In line with the literature, we find that when CBDC and deposits are perfect substitutes, as long as they have the same resource cost per unit of effective real balances, the central bank can offer bank loans at a loan rate that renders the non-competitive banks indifferent to the introduction of the CBDC and CBDC has no real effects on the economy. Additionally, it is crucial to account for the collateral requirement that banks must respect when borrowing from the central bank, as the central bank lending rate depends on how restrictive the collateral constraint is. The tighter the constraint is (the lower the fraction of banks' bond holdings that can be pledged as collateral), the lower the central bank loan rate should be to keep the equilibrium allocations unchanged when introducing a CBDC.

However, different from the results in the literature, we show that while the central bank compensates for the lower deposit funding by lending to banks, to meet collateral requirements banks reduce capital investments and the central bank assumes some of the credit extension role to firms, which is undesirable. In other words, the central bank insulates bank profits but not the bank's business model. It follows that, although the CBDC introduction has no real effects on the economy, it does not guarantee full neutrality as it affects the bank's business model and the actors' role in the economy

The results from the dynamic model extension show that an increase in CBDC issuance does not lead to bank disintermediation or crowding out of deposits, but it expands bank activity as it raises banks' capital investments, but reduces aggregate capital and production as households reduce direct capital investments and raise consumption.

Overall, our findings help policymakers and central bankers design and implement CBDCs to minimize the risk of bank disintermediation. A possible extension in the analysis of the equivalence result is to investigate the transition from the equilibrium with no CBDC to the equilibrium after CBDC has been introduced and identify the driving forces governing the transition between the two.

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A Condition under which the collateral constraint binds

Assuming interior solutions, the bank's optimality conditions for loans and bonds are, respectively:

$$\mathbb{E}_t\left[\Lambda_{t+1}(R^k_{t+1} - R^l_{t+1} - l_{t+1}\frac{\partial R^l_{t+1}}{\partial l_{t+1}})\right] = \gamma_t\left(1 + \theta_b \frac{b_{t+1}}{{R^l}^2_{t+1}}\frac{\partial R^l_{t+1}}{\partial l_{t+1}}\right),$$
$$\mathbb{E}_t\left[\Lambda_{t+1}(R^k_{t+1} - R^b_{t+1})\right] = \gamma_t \frac{\theta_b}{R^l_{t+1}},$$

where γ_t denotes the Lagrange multiplier associated with the collateral constraint. Subtracting the condition for bonds from the one for loans:

$$\mathbb{E}_t\left[\Lambda_{t+1}(R^b_{t+1} - R^l_{t+1} - l_{t+1}\frac{\partial R^l_{t+1}}{\partial l_{t+1}})\right] = \gamma_t\left(1 - \frac{\theta_b}{R^l_{t+1}} + \theta_b \frac{b_{t+1}}{{R^l}^2_{t+1}}\frac{\partial R^l_{t+1}}{\partial l_{t+1}}\right). \quad (43)$$

To define the sign of the RHS, recall that $\theta_b \in [0, 1]$, and since the rate of return on reserves is positive, and we assumed interior solutions, all the terms are positive.

We define the elasticity of central bank's loans with respect to their rate of returns as

$$\eta_{l,t+1} = \frac{\partial l_{t+1}}{\partial R^l_{t+1}}\frac{R^l_{t+1}}{l_{t+1}},$$

such that we can rewrite the last term on the LHS as

$$\frac{1}{\eta_{l,t+1}}R^l_{t+1}.$$

Expression (43) says that the collateral constraint is binding if:

$$R^b_{t+1} - R^l_{t+1} > \frac{1}{\eta_{l,t+1}}R^l_{t+1}.$$

Rearranging

$$R^b_{t+1} > R^l_{t+1} + \frac{1}{\eta_{l,t+1}}R^l_{t+1}.$$

We can conclude that the collateral constraint is binding if the sum of the cost of borrowing from the central bank and the bank's cost of taking on more loans is cheaper than the return the bank gets from holding government bonds:[16]

$$\gamma_t > 0, \qquad l_{t+1} = \theta_b \frac{b_{t+1}}{R^l_{t+1}} \qquad \text{iff } R^b_{t+1} > R^l_{t+1} + \frac{1}{\eta_{l,t+1}} R^l_{t+1}.$$

B Equivalence

For convenience, we repeat Proposition 1 as in Section 3 and prove it formally.

Proposition 1. *Consider a policy that implements an equilibrium with deposits, reserves, central bank loans, and government bonds, where the latter are used as banks' collateral to obtain central bank loans. The central bank introduces a new payment instrument, CBDC, which is a perfect substitute for deposits. There exists another policy and equilibrium, indicated by circumflexes. with less deposits and reserves, a positive amount of CBDC, central bank loans, and government bonds, a different ownership structure of capital, additional taxes on the household, and otherwise the same equilibrium allocation and price system.*
■

Suppose that deposit holdings decrease by a magnitude of Δ from the initial equilibrium, i.e. $\hat{n}_{t+1} - n_{t+1} = -\Delta$. Suppose also that real balances, the aggregate capital stock and the reserves-to-deposits ratio remain unchanged in the new equilibrium, i.e.,

$$\hat{z}_{t+1} = z_{t+1}, \quad \hat{k}_{t+1} = k_{t+1}, \quad \hat{\zeta}_{t+1} = \zeta_{t+1}.$$

The above implies the following changes in the other equilibrium quantities:

$$\begin{aligned} \hat{m}_{t+1} - m_{t+1} &= \frac{1}{\lambda}\Delta, & \hat{r}_{t+1} - r_{t+1} &= -\zeta_{t+1}\Delta, \\ \hat{l}_{t+1} - l_{t+1} &= (1-\zeta_{t+1})\Delta, & \hat{b}_{t+1} - b_{t+1} &= \frac{\hat{l}_{t+1} R^l_{t+1}}{\theta_b}, \\ \hat{k}^h_{t+1} - k^h_{t+1} &= \left(1 - \frac{1}{\lambda}\right)\Delta, & \hat{k}^g_{t+1} - k^g_{t+1} &= -\left(1 - \frac{1}{\lambda}\right)\Delta + \hat{b}_{t+1}, \end{aligned}$$

where l_{t+1} and b_{t+1} will be normalized to zero in what follows.[17]

[16] We replicated the same analysis in the setting by Burlon, Muñoz, and Smets (2024), and we got an analogous result.

[17] To guarantee the non-negativity of deposits, capital holdings, and reserves, Δ must not be too large. Specifically, we impose

$$\Delta \leq n_{t+1}, \qquad \zeta_{t+1}\Delta \leq r_{t+1}, \qquad \left(1 - \frac{1}{\lambda}\right)\Delta \leq k^g_{t+1}, \qquad \left(1 - \frac{1}{\lambda}\right)\Delta \geq -k^h_{t+1}.$$

First, we show that the new policy has no real effects on the economy, given an appropriate level of interest rate on central bank loans. Note that, before the implementation of the new policy, the cash flows generated in the first and second periods of the bank's operations are given by equation (10) and (11), respectively. Recalling that in equilibrium $\zeta_{t+1} = \bar{\zeta}_{t+1}$, the changes in bank profits at dates t and $t+1$ are, respectively:

$$\hat{\Pi}^b_{1,t} - \Pi^b_{1,t} = \Delta \left(\nu(\zeta_{t+1}, \zeta_{t+1})\right), \tag{44}$$

$$\hat{\Pi}^b_{2,t+1} - \Pi^b_{2,t+1} = \Delta \left(R^n_{t+1} - \zeta_{t+1} R^r_{t+1} - (1-\zeta_{t+1}) \left(1 + \frac{R^k_{t+1} - R^b_{t+1}}{\theta_b}\right) R^l_{t+1} \right). \tag{45}$$

Let $\hat{T}_{1,t}$ be a tax on the household at date t that compensates for the reduced bank losses:

$$\hat{T}_{1,t} = \hat{\Pi}^b_{1,t} - \Pi^b_{1,t} = \Delta \left(\nu(\zeta_{t+1}, \zeta_{t+1})\right). \tag{46}$$

We denote $\hat{T}_{2,t+1}$ as a tax at date $t+1$ that compensates for the change in the household's portfolio return as well as for the change in bank profits that the household collects at date $t+1$:[18]

$$\begin{aligned} \hat{T}_{2,t+1} &= (\hat{k}^h_{t+1} - k^h_{t+1}) R^k_{t+1} + (\hat{n}_{t+1} - n_{t+1}) R^n_{t+1} + (\hat{m}_{t+1} - m_{t+1}) R^m_{t+1} + \hat{\Pi}^b_{2,t+1} - \Pi^b_{2,t+1} \\ &= \Delta \left[\left(1 - \frac{1}{\lambda}\right) R^k_{t+1} + \frac{R^m_{t+1}}{\lambda} - \zeta_{t+1} R^r_{t+1} - (1-\zeta_{t+1}) \left(1 + \frac{R^k_{t+1} - R^b_{t+1}}{\theta_b}\right) R^l_{t+1} \right]. \end{aligned} \tag{47}$$

Let $\mathscr{T}_t = \hat{T}_{1,t} + \mathbb{E}_t \Lambda_{t+1} \hat{T}_{2,t+1}$ denote the market value of taxes at date t. Substituting the two expressions for taxes, equations (46) and (47), and using conditions from the household's optimization problem, we can rewrite $\mathscr{T}_t$ as

$$\mathscr{T}_t = \Delta \left[\nu(\zeta_{t+1}, \zeta_{t+1}) + \frac{R^n_{t+1} - \zeta_{t+1} R^r_{t+1} - (1-\zeta_{t+1}) \left(1 + \frac{R^k_{t+1} - R^b_{t+1}}{\theta_b}\right) R^l_{t+1}}{R^f_{t+1}} \right].$$

In order for the new policy to have no real effects on the economy, the market value of taxes must be zero. This is true if the central bank posts a loan rate equal to equation (26):

$$R^l_{t+1} = \frac{R^n_{t+1} + \nu(\zeta_{t+1}, \zeta_{t+1}) R^f_{t+1} - \zeta_{t+1} R^r_{t+1}}{(1-\zeta_{t+1}) \left(1 + \frac{R^k_{t+1} - R^b_{t+1}}{\theta_b}\right)}.$$

[18] Given the household's trades-off between CBDC and deposits, expression (9), it follows that: $\frac{\lambda}{R^m_{t+1}} = R^n_{t+1} - (1 - \frac{1}{\lambda}) R^f_{t+1}$.

We denote the market value of the changes in bank profits at date t as $\mathscr{P}_t = (\hat{\Pi}^b_{1,t} - \Pi^b_{1,t}) + \mathbb{E}_t\Lambda_{t+1}(\hat{\Pi}^b_{2,t+1} - \Pi^b_{2,t+1})$. Plugging in the expressions for changes in bank profits, equations (44) and (45), and using the definition of the risk-free rate from the household's problem, $\mathscr{P}_t$ reads

$$\begin{aligned}\mathscr{P}_t &= \Delta\left(\nu(\zeta_{t+1}, \zeta_{t+1})\right) \\ &+ \frac{1}{R^f_{t+1}}\Delta\left(R^n_{t+1} - \zeta_{t+1}R^r_{t+1} - (1-\zeta_{t+1})\left(1 + \frac{R^k_{t+1} - R^b_{t+1}}{\theta_b}\right)R^l_{t+1}\right),\end{aligned}$$

which is equal to zero given equation (26). It follows that if the central bank offers an interest rate on central bank loans according to equation (26), the market values of the taxes and of the changes in bank profits are zero.

Next, we show that the government's dynamic and intertemporal budget constraints continue to be satisfied with the new policy. Before the implementation of the new policy, the government budget constraint at time t reads:

$$k^g_{t+1} - m_{t+1} - r_{t+1} = k^g_t R^k_t - m_t R^m_t - r_t R^r_t + \tau_t - m_{t+1}\mu - r_{t+1}\rho. \tag{48}$$

The government budget constraint at time t with the new policy and changes is

$$\begin{aligned}\hat{k}^g_{t+1} + \hat{l}_{t+1} - \hat{m}_{t+1} - \hat{r}_{t+1} - \hat{b}_{t+1} = k^g_t R^k_t - m_t R^m_t - r_t R^r_t + \tau_t \\ - \hat{m}_{t+1}\mu - \hat{r}_{t+1}\rho + \hat{T}_{1,t}.\end{aligned}$$

Rearranging, simplifying, and collecting terms:

$$\begin{aligned}k^g_{t+1} - m_{t+1} - r_{t+1} + \Delta\left(\frac{\mu^m}{\lambda} - (\nu(\zeta_{t+1}, \zeta_{t+1}) + \rho\zeta_{t+1})\right) = k^g_t R^k_t - m_t R^m_t - r_t R^r_t + \tau_t \\ - m_{t+1}\mu - r_{t+1}\rho.\end{aligned} \tag{49}$$

The government budget constraints before and after the intervention at time t, equations (48) and (49), are identical as long as

$$\frac{\mu}{\lambda} = \nu(\zeta_{t+1}, \zeta_{t+1}) + \zeta_{t+1}\rho.$$

That is, when the public and private sectors are equally efficient in providing liquidity to the household, the government's budget constraint at date t is unaffected by the changes in allocation

Similarly, before the new policy, the government budget constraint at time $t+1$ reads

$$\begin{aligned} k^g_{t+2} - m_{t+2} - r_{t+2} = & \; k^g_{t+1} R^k_{t+1} - m_{t+1} R^m_{t+1} - r_{t+1} R^r_{t+1} \\ & + \tau_{t+1} - n_{t+2}\theta_{t+1} - m_{t+2}\mu - r_{t+2}\rho. \end{aligned} \tag{50}$$

With the new policy and changes, the government budget constraint at time $t+1$ becomes

$$\begin{aligned} k^g_{t+2} - m_{t+2} - r_{t+2} = & \; \hat{k}^g_{t+1} R^k_{t+1} + \hat{l}_{t+1} R^l_{t+1} - \hat{m}_{t+1} R^m_{t+1} - \hat{r}_{t+1} R^r_{t+1} - \hat{b}_{t+1} R^b_{t+1} \\ & + \tau_{t+1} - n_{t+2}\theta_{t+1} - m_{t+2}\mu - r_{t+2}\rho + \hat{T}_{2,t+1}. \end{aligned}$$

Rearranging, simplifying, and collecting terms:

$$\begin{aligned} k^g_{t+2} - m_{t+2} - r_{t+2} = & \hat{k}^g_{t+1} R^k_{t+1} + \hat{l}_{t+1} R^l_{t+1} - \hat{m}_{t+1} R^m_{t+1} - \hat{r}_{t+1} R^r_{t+1} - \hat{b}_{t+1} R^b_{t+1} \\ & + \tau_{t+1} - n_{t+2}\theta_{t+1} - \hat{r}_{t+1} R^r_{t+1} - m_{t+2}\mu - r_{t+2}\rho + \hat{T}_{2,t+1}. \end{aligned} \tag{51}$$

Using the expression for the central bank loan rate we derived, equation (26), it follows that the government budget constraints before and after the intervention at time $t+1$, equations (50) and (51), are the same. In other words, the central bank loan rate ensuring that the market values of taxes and changes in bank profits are zero, also ensures that the government budget constraint at time $t+1$ is unaffected by the changes in allocation.

We claimed initially that the proposed intervention does not change the price system. In this case, the firm's optimal production decisions and profits are unchanged. Lastly, we must show that the modified bank's portfolio is still optimal. Before the intervention, the bank's choice set is determined by the cost function, the household's stochastic discount factor, rates on returns on capital and reserves, and the deposit funding schedule. The new policy leaves unchanged the cost function, the stochastic discount factor, and the rates on returns on capital and reserves. After the intervention, as the household holds more CBDC, there is a modified deposit funding schedule, together with a central bank loan funding schedule. The central bank needs to post an appropriate loan funding schedule to induce the non-competitive bank to go along with the equivalent balance sheet positions as before the intervention. Subject to this schedule, the bank chooses loans that make up for the reduction in funding from the household, net of reserves, at the same effective price. The central bank chooses a loan funding schedule which mirrors the deposit funding schedule and posts the loan rate as in equation (26).

C Equivalence with imperfect substitutability between payment instruments

In Section 3 we consider CBDC and deposits as perfectly substitutable for the household. However, some works in the literature consider imperfect substitutability between the two instruments [see, e.g., Agur, Ari, and Dell'Ariccia (2022), Bacchetta and Perazzi (2022), Barrdear and Kumhof (2022), Burlon, Muñoz, and Smets (2024) and Kumhof and Noone (2021)]. We now assume a constant elasticity of substitution (CES) functional form for the household's real balances:

$$z_{t+1}(m_{t+1}, n_{t+1}) = \left(\lambda m_{t+1}^{1-\epsilon} + n_{t+1}^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}},$$

where $\lambda \geq 0$ represents the benefits of CBDC relative to deposits, and $\epsilon \geq 0$ is the inverse elasticity of substitution between payment instruments.

Proposition 2. *Consider a policy that implements an equilibrium with deposits, reserves, central bank loans, and government bonds, where the latter are used as banks' collateral to obtain central bank loans. The central bank introduces a new payment instrument, CBDC, which is an imperfect substitute for deposits. There does not exist another policy and equilibrium, indicated by circumflexes, that guarantees the same equilibrium allocation and price system.*
■

Suppose again that deposit holdings decrease by a magnitude of Δ from the initial equilibrium and that real balances, the aggregate capital stock and the reserves-to-deposits ratio remain unchanged in the new equilibrium. This implies the same changes in equilibrium quantities as we have seen in Appendix B, except for the CBDC. Due to the imperfect substitutability between CBDC and deposits, in order for real balances to remain unchanged, the quantity of CBDC must change according to

$$\hat{m}_{t+1} - m_{t+1} = \left[\frac{1}{\lambda}\left(n_{t+1}^{1-\epsilon} - \hat{n}_{t+1}^{1-\epsilon}\right) + m_{t+1}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} - m_{t+1}.$$

We define taxes at dates t and $t+1$, equations (46) and (47) respectively, as in Appendix B, as well as the market values of taxes, $\mathscr{T}_t = \hat{T}_{1,t} + \mathbb{E}_t \Lambda_{t+1} \hat{T}_{2,t+1}$. The central bank loan rate ensuring that the market value of taxes is zero is:[19]

$$R^l_{t+1} = \frac{\mathscr{A}_t R^n_{t+1} - \zeta_{t+1} R^r_{t+1} + \left(\nu(\zeta_{t+1}, \zeta_{t+1}) + 1 - \mathscr{A}_t\right) R^f_{t+1}}{(1-\zeta_{t+1})\left(1 + \frac{R^k_{t+1} - R^b_{t+1}}{\theta_b}\right)}, \tag{52}$$

where

$$\mathscr{A}_t = \lambda \left(\frac{\hat{m}_{t+1} - m_{t+1}}{\Delta}\right)\left(\frac{n_{t+1}}{m_{t+1}}\right)^{\epsilon}.$$

Consider the changes in bank profits at dates t and $t+1$ as given from equations (44) and (45), respectively. We can check whether the market value of the changes in bank profits, $\mathscr{P}_t = (\hat{\Pi}^b_{1,t} - \Pi^b_{1,t}) + \mathbb{E}_t \Lambda_{t+1}(\hat{\Pi}^b_{2,t+1} - \Pi^b_{2,t+1})$, also reduces to zero given the central bank loan rate in expression (52). It turns out this is not true. In particular, after making the appropriate substitutions, the market value of changes in bank profits reads:

$$\mathscr{P}_t = \mathbb{E}_t \frac{1}{R^f_{t+1}} \Delta \left(R^n_{t+1} - \mathscr{A}_t R^n_{t+1} - (1-\mathscr{A}_t) R^f_{t+1}\right).$$

Notice that if there were perfect substitutability between CBDC and deposits (i.e., $\epsilon = 0$), as in the case studies in Section 3, $\mathscr{A}_t$ equals 1, and the market value of the changes in bank profits reduces to zero. It follows that, in case of imperfect substitutability between CBDC and deposits, the central bank lending rate that renders the market value of taxes zero does not result in changes in bank profits being zero. In other words, the central bank cannot make the bank indifferent to the competition from CBDC. In fact, a change in the bank's profitability implies that the new policy does not guarantee the same equilibrium allocation as before, implying that the introduction of CBDC has real effects on the economy. In Brunnermeier and Niepelt (2019), one condition for equivalence to hold is that CBDC and deposits are minimally substitutable, such that their marginal liquidity contribution is unchanged. When the two instruments are imperfect substitutes, as in the case under study, the marginal rate of substitution is not constant, and equivalence is not guaranteed.

[19] When deriving the central bank loan rate, we use expression (9) for the household's trade-off between CBDC and deposits. Given the CES functional form assumption, equation (24) it follows that: $\frac{m_{t+1}}{n_{t+1}} = \left(\lambda \frac{\chi^n_{t+1}}{\chi^m_{t+1}}\right)^{\frac{1}{\epsilon}}$.

D Additional figures

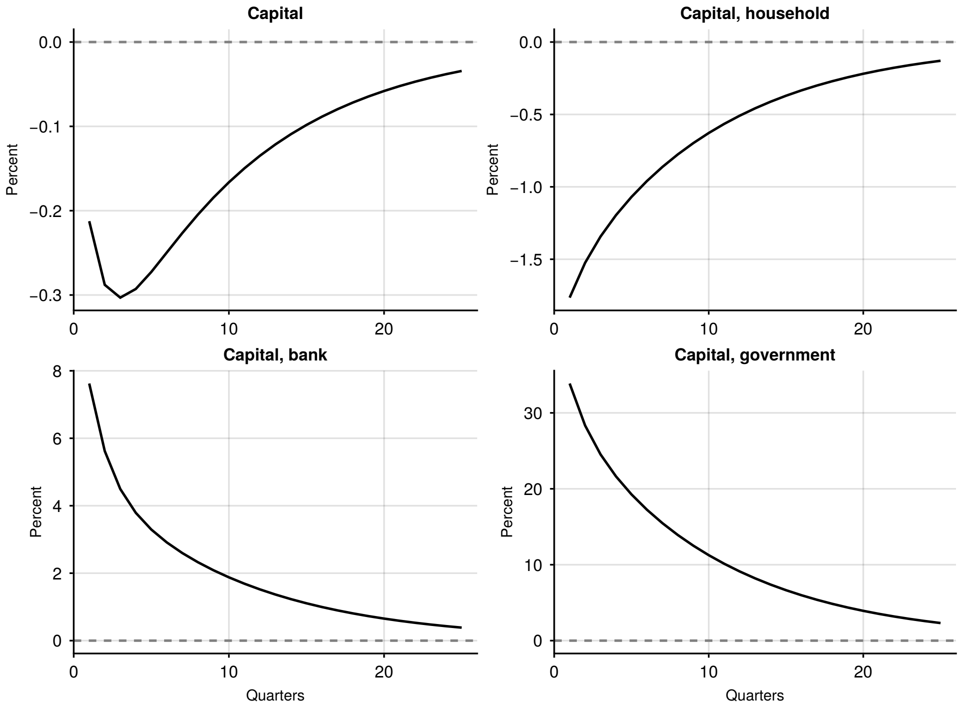

Figure 3: Capital response breakdown by components

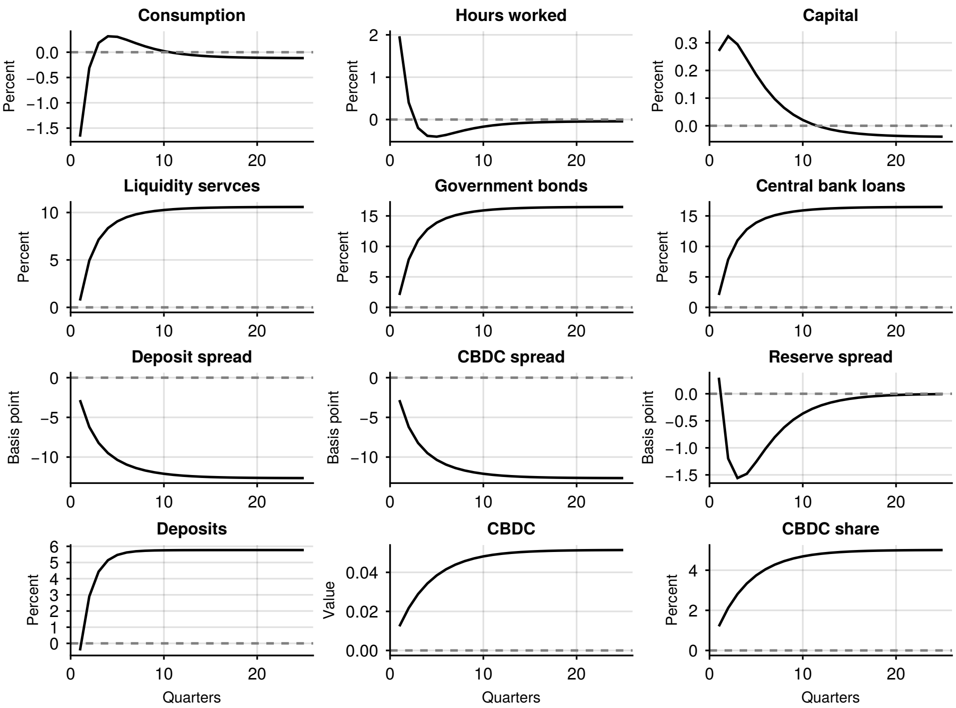

Figure 4: Impulse responses to a permanent increase in CBDC share from zero to 5%

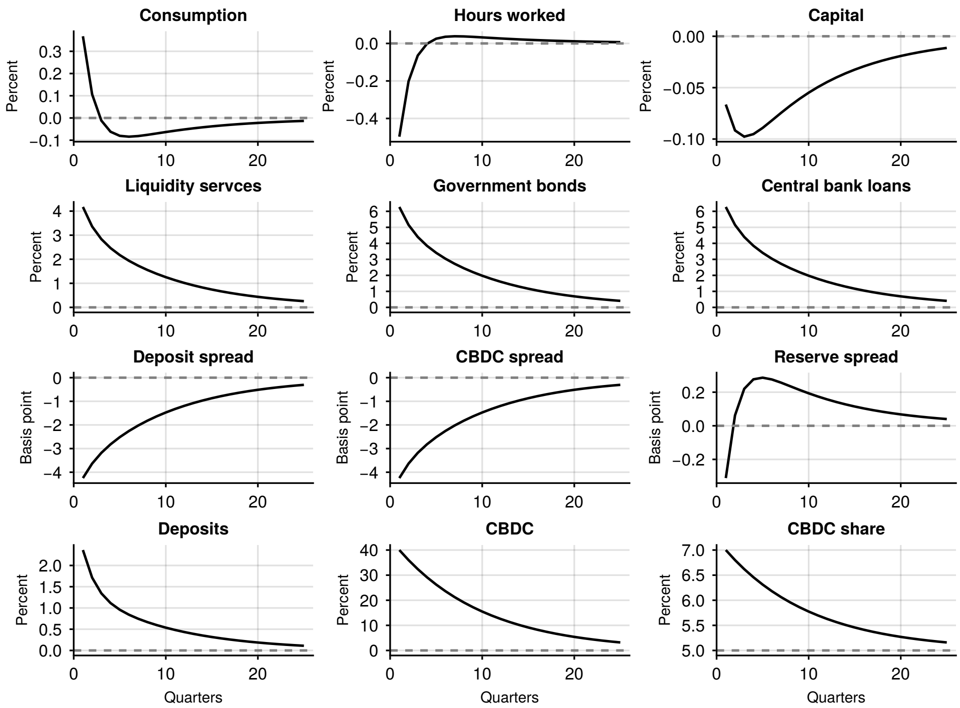

Figure 5: Impulse responses to a temporary increase in CBDC share from 5% to 7%

D.1 Robustness

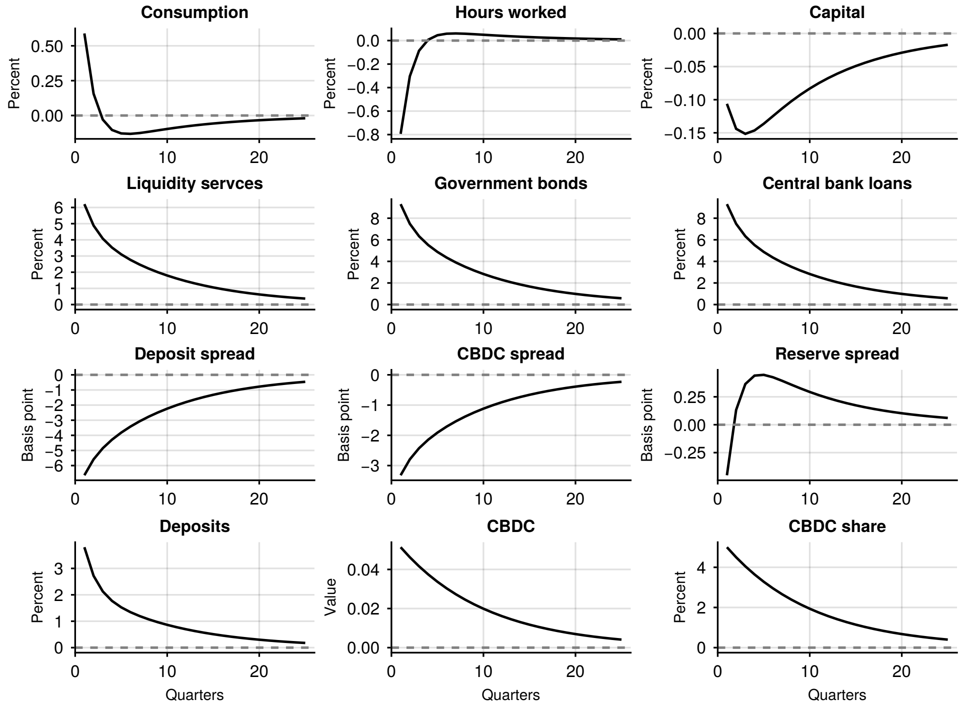

Figure 6: Impulse responses to a temporary increase in CBDC share with lower relative benefit of CBDC ($\lambda = 0.5$)

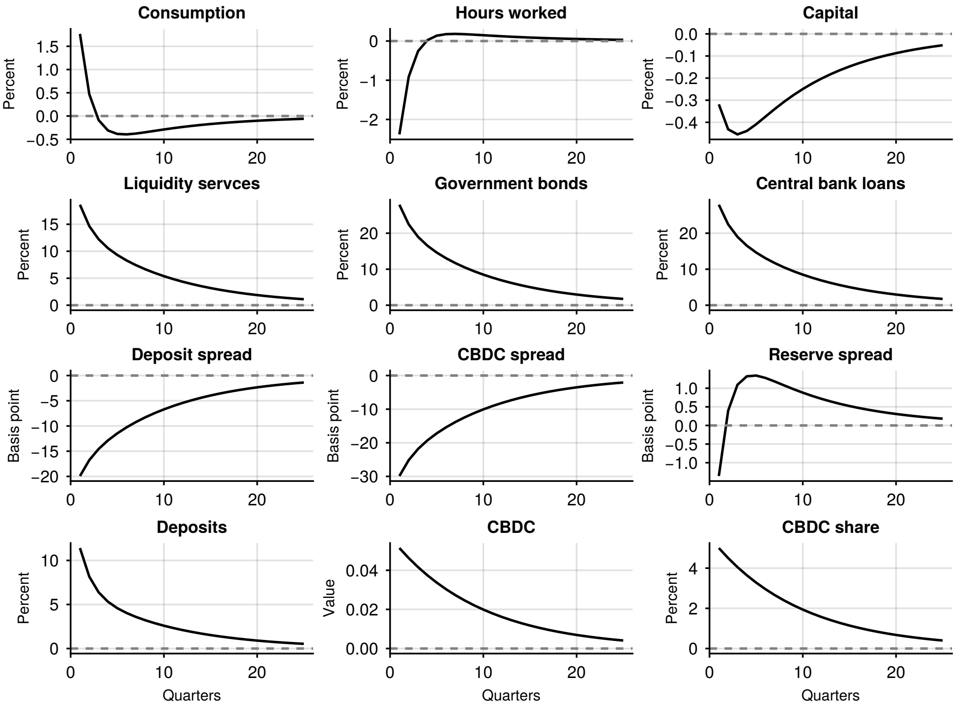

Figure 7: Impulse responses to a temporary increase in CBDC share with higher relative benefit of CBDC ($\lambda = 1.5$)

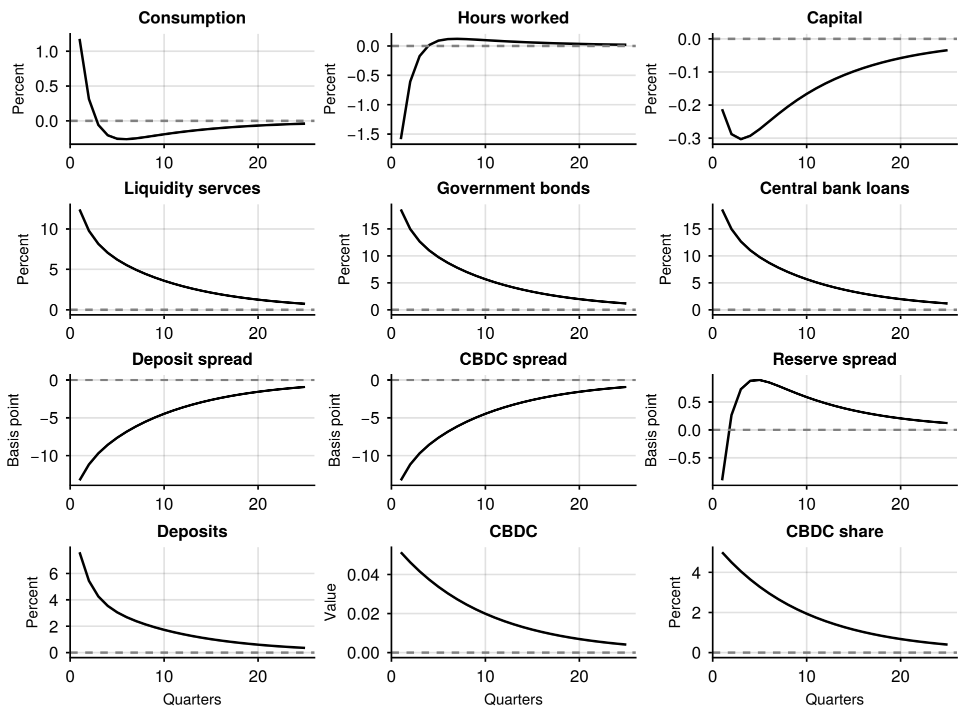

Figure 8: Impulse responses to a temporary increase in CBDC share with a lower haircut on bonds ($\theta_b = 0.999$)

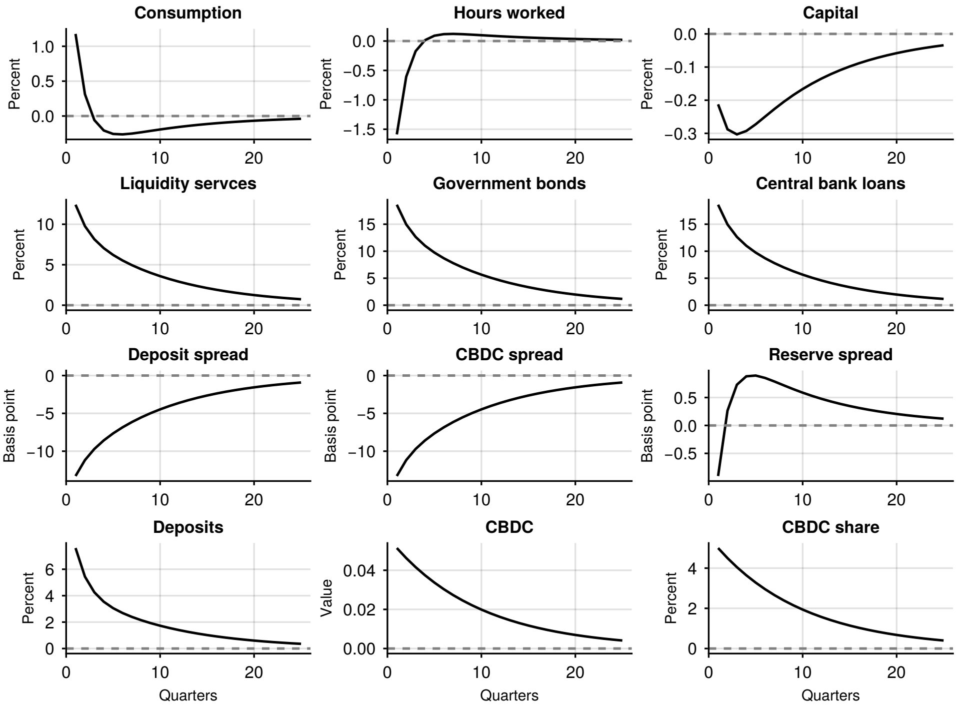

Figure 9: Impulse responses to a temporary increase in CBDC share with a higher haircut on bonds ($\theta_b = 0.985$)